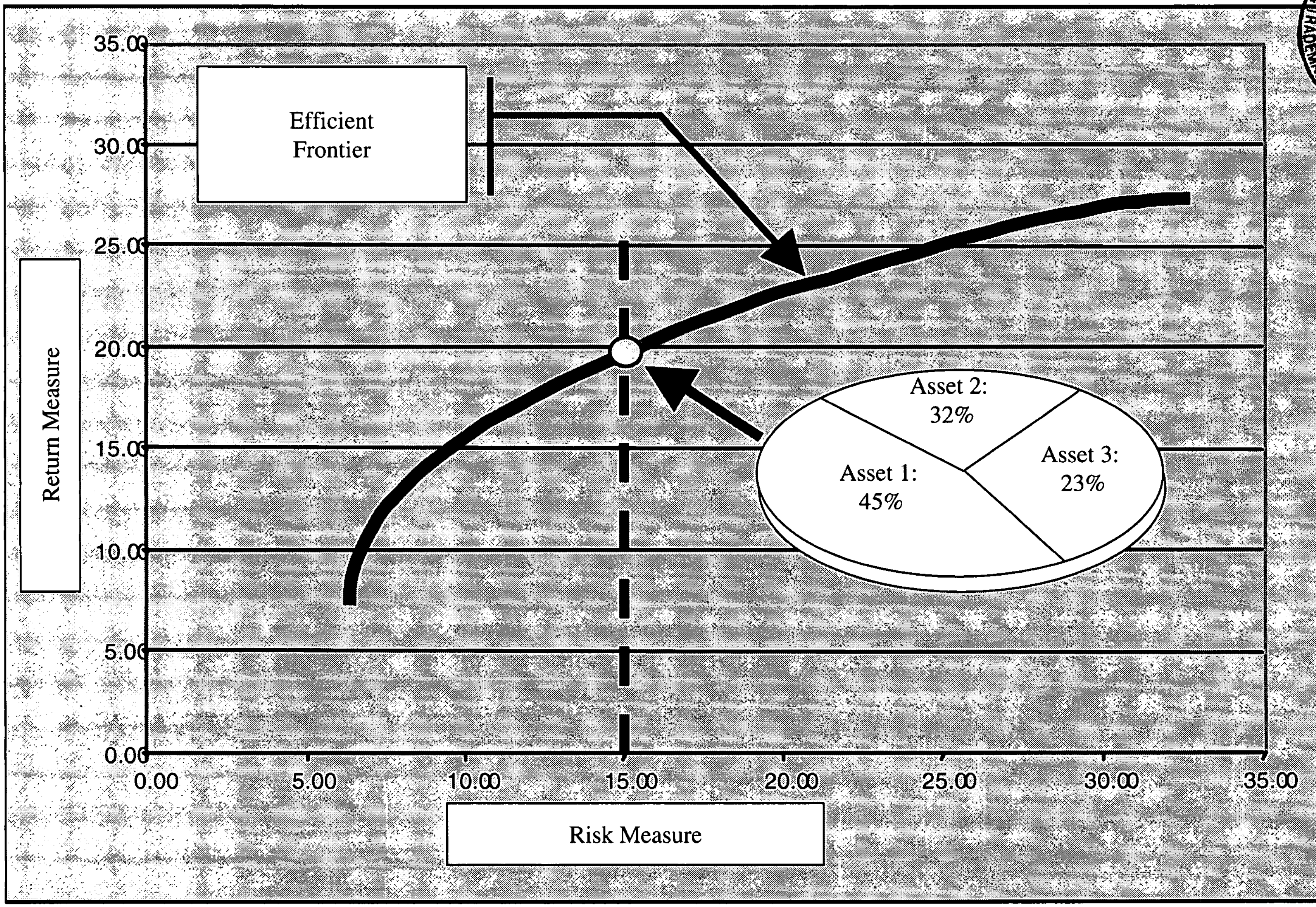
Fig. 1

BACKGROUND ART



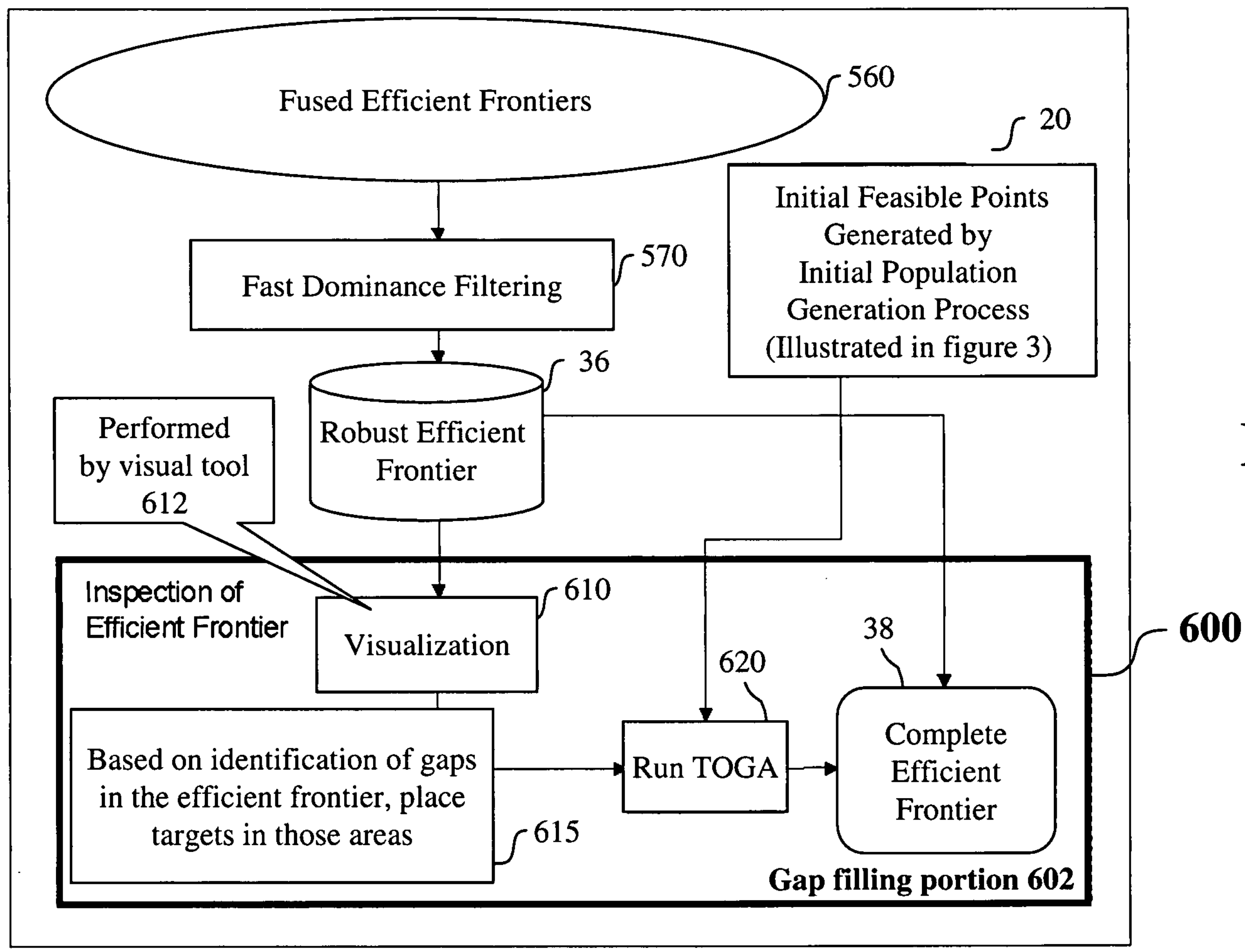
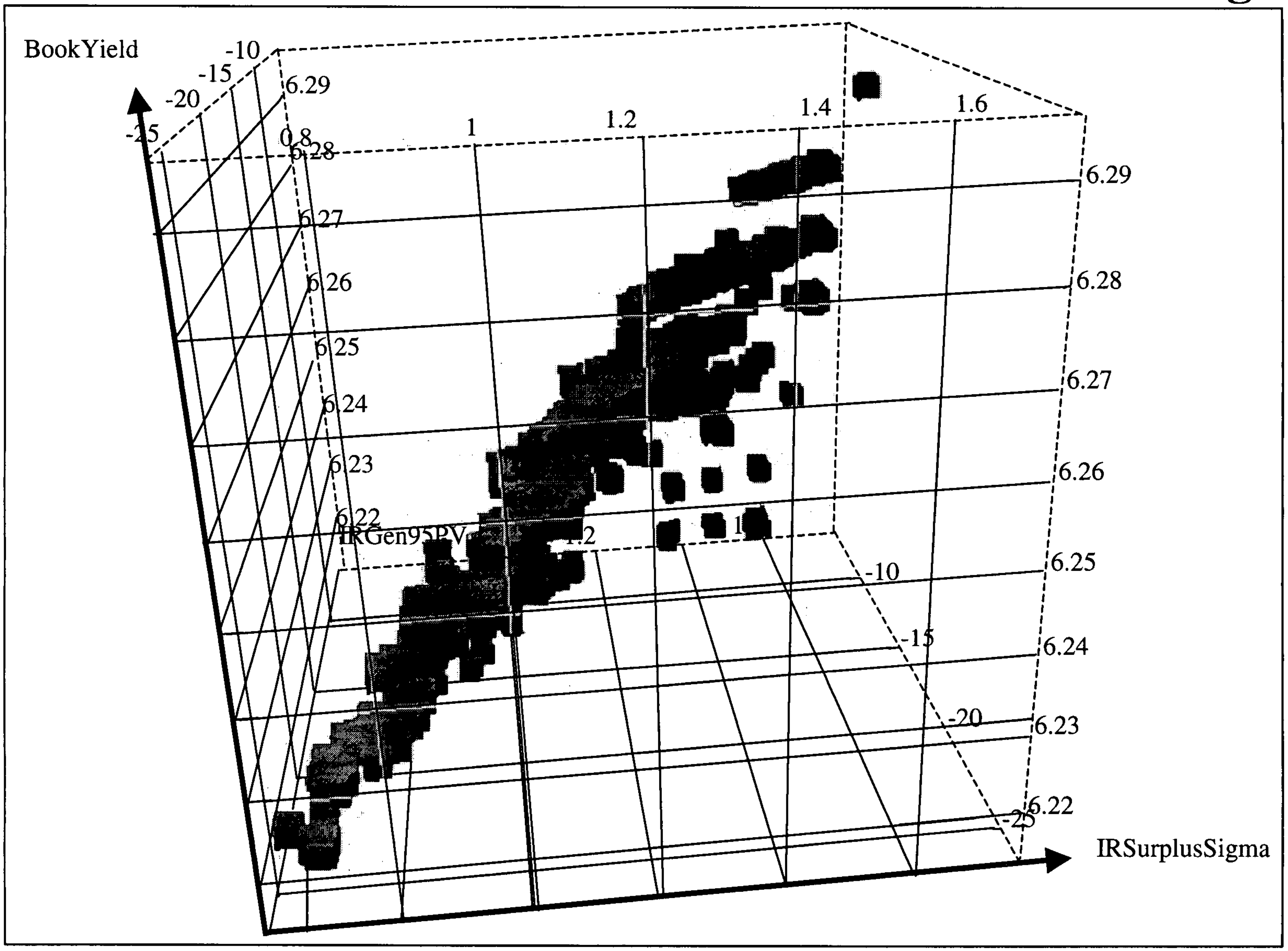


Fig. 12

Process to interactively fill any gaps in the identified efficient frontier

Fig. 13

Efficient Frontier in a 3D View



EXAMPLE OF PARALLEL COORDINATE PLOT

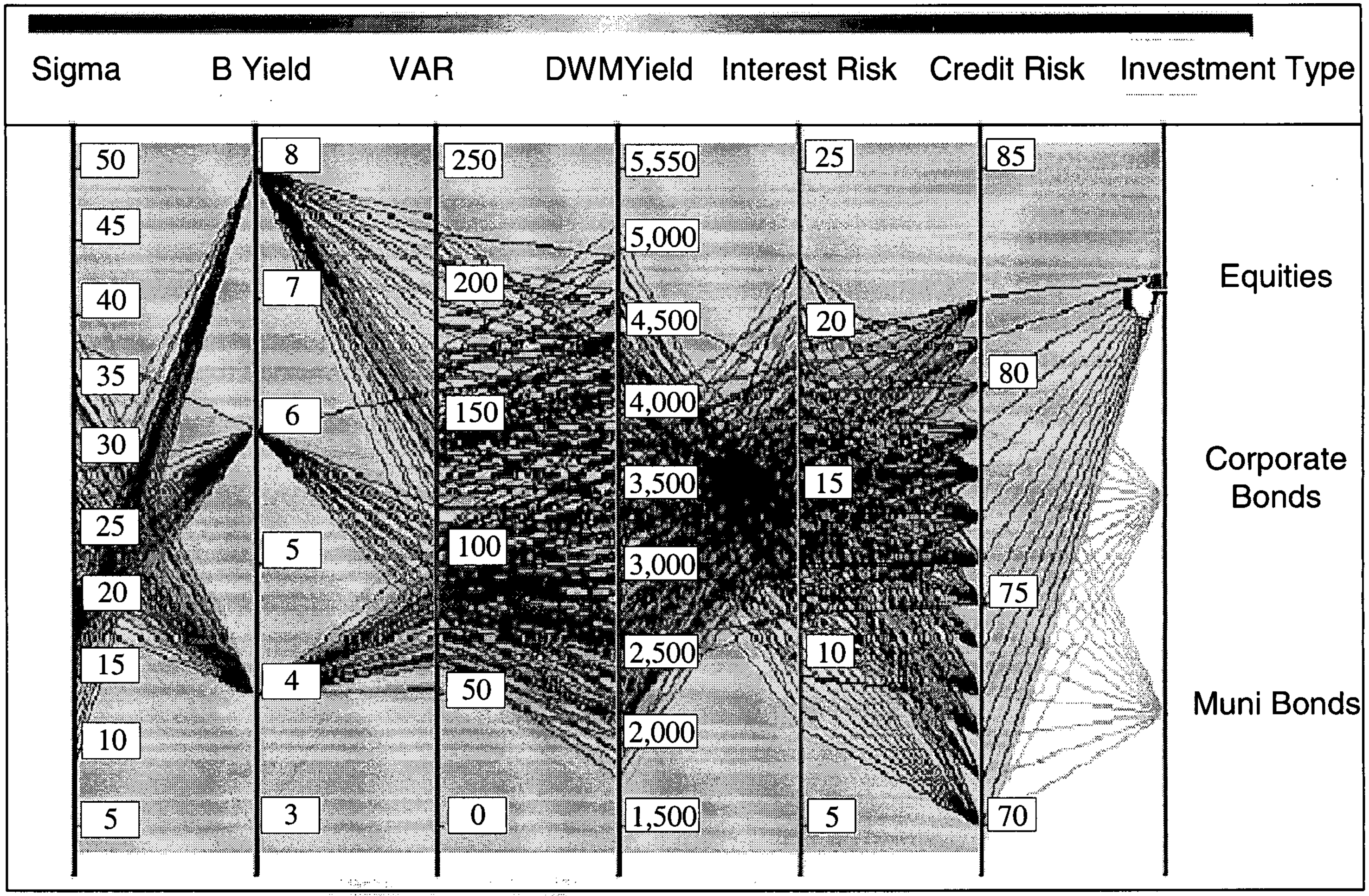
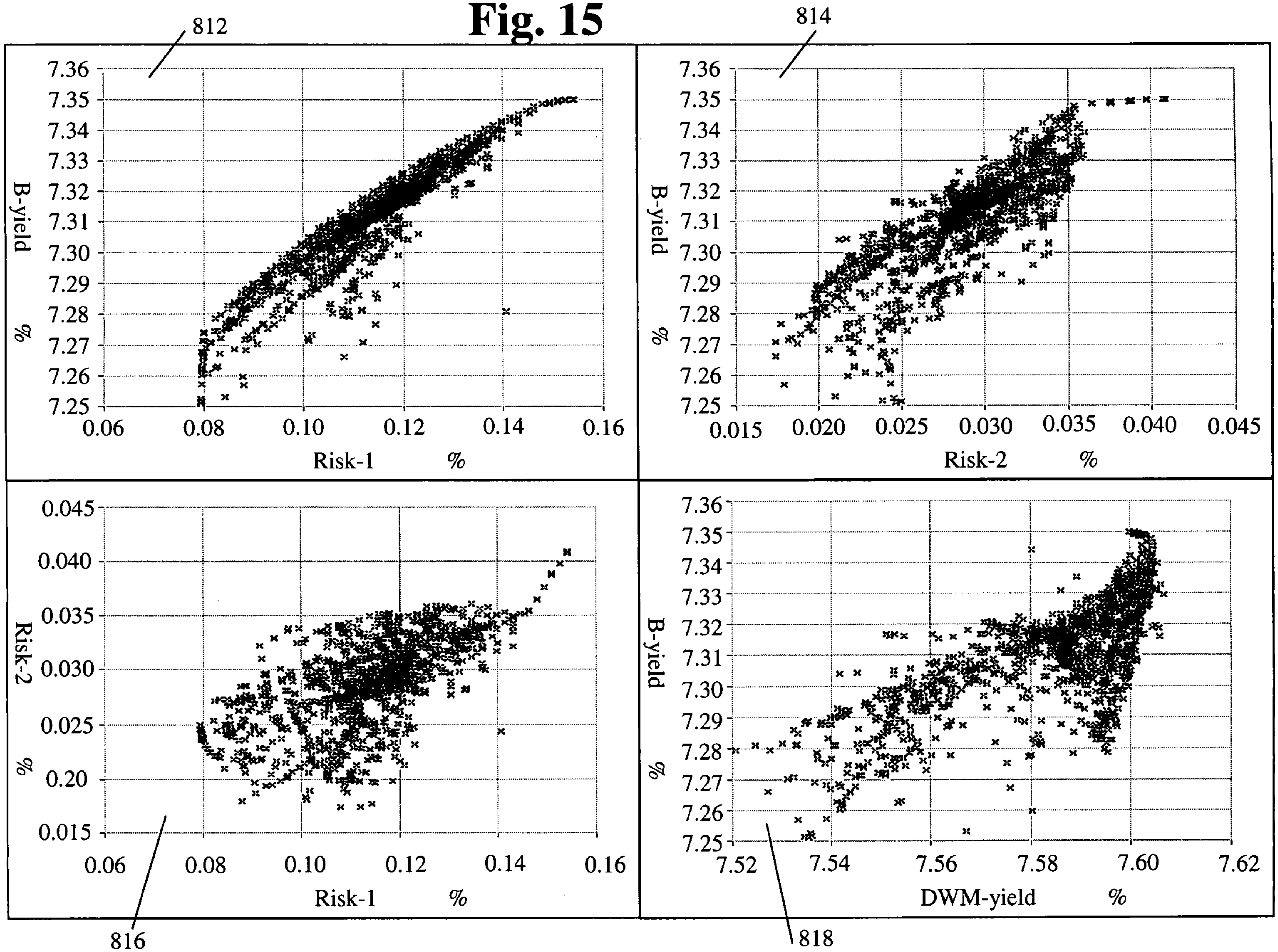
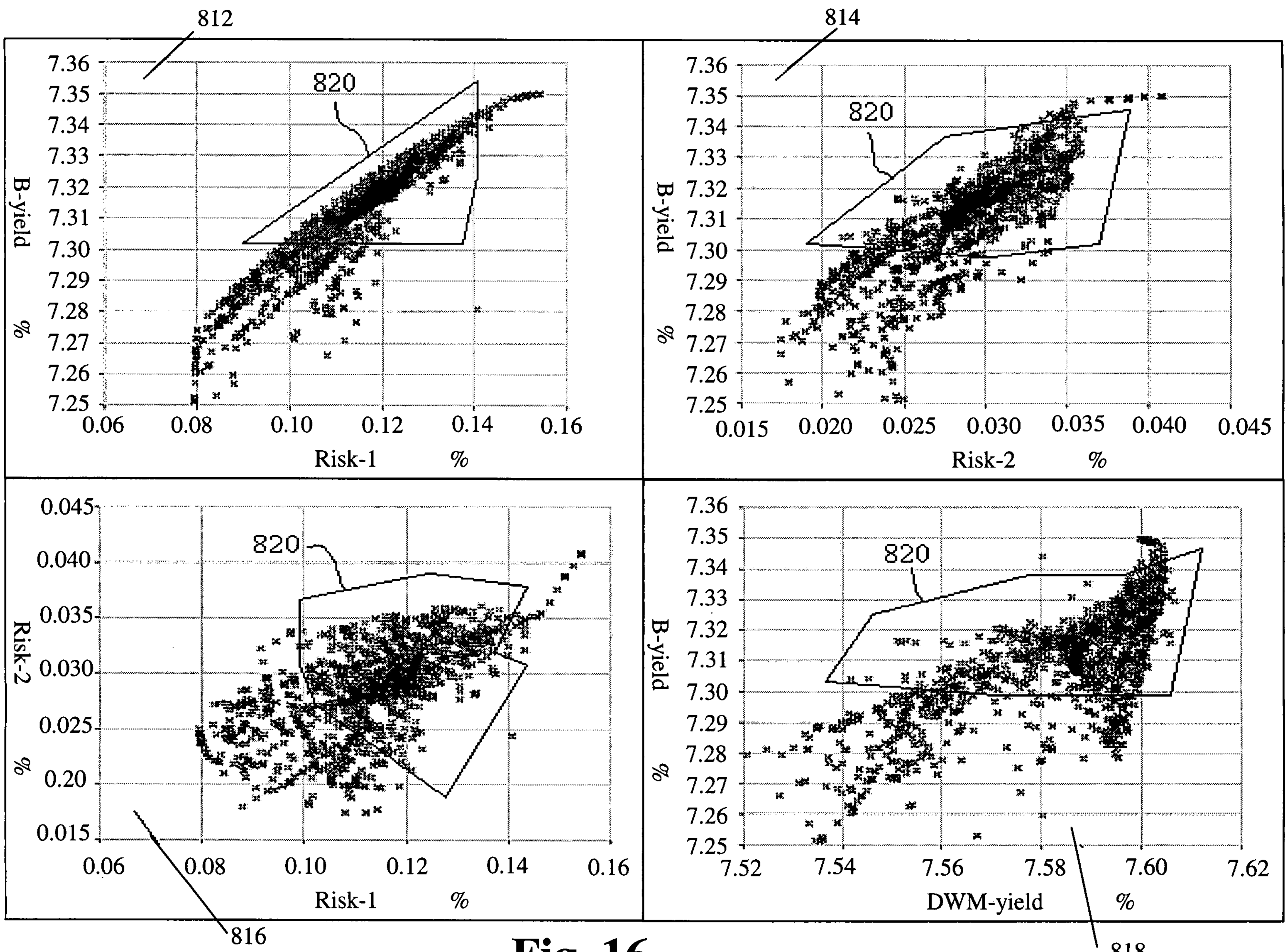


Fig. 14

Fig. 15


812
814
816
818
B-yield %
Risk-1 %
Risk-2 %
DWM-yield %
7.36
7.35
7.34
7.33
7.32
7.31
7.30
7.29
7.28
7.27
7.26
7.25
0.06
0.08
0.10
0.12
0.14
0.16
0.015
0.020
0.025
0.030
0.035
0.040
0.045
0.20
7.52
7.54
7.56
7.58
7.60
7.62



812
814
816
818
820
820
820
820
B-yield %
Risk-1 %
Risk-2 %
B-yield %
Risk-2 %
Risk-1 %
B-yield %
DWM-yield %
7.36
7.35
7.34
7.33
7.32
7.31
7.30
7.29
7.28
7.27
7.26
7.25
0.06
0.08
0.10
0.12
0.14
0.16
0.015
0.020
0.025
0.030
0.035
0.040
0.045
0.20
7.52
7.54
7.56
7.58
7.60
7.62

Fig. 16

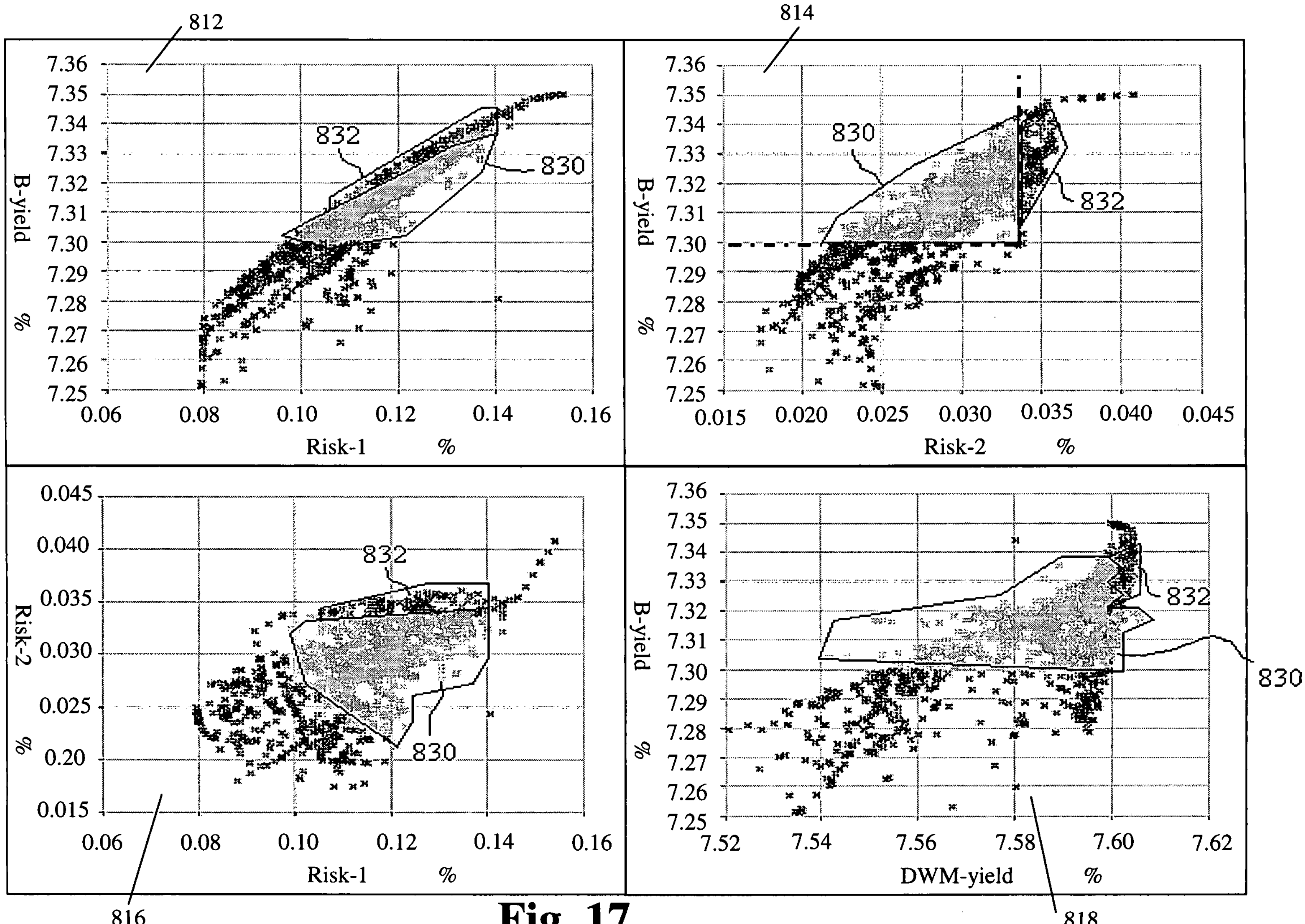
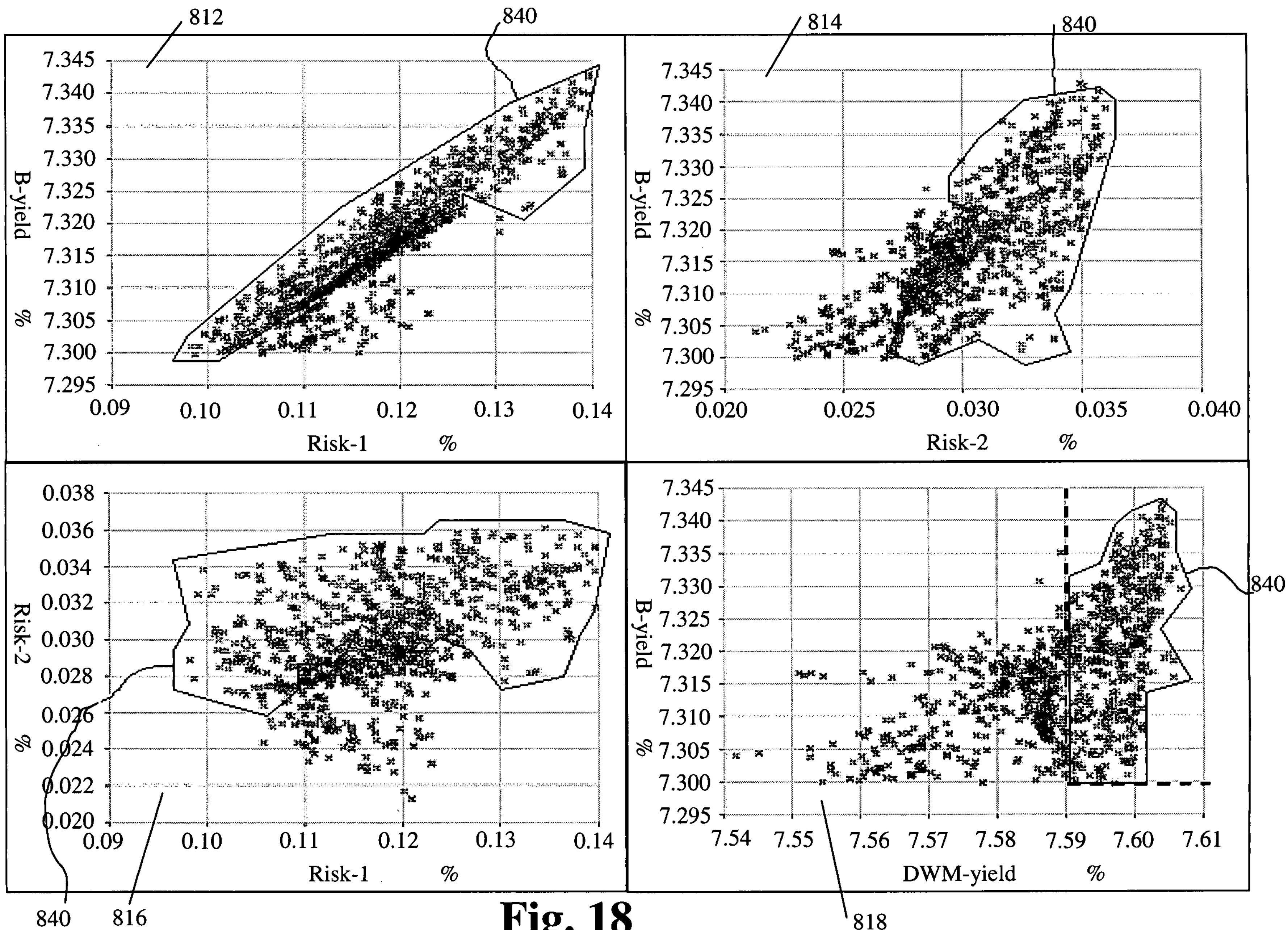

812
814
816
818
B-yield %
Risk-1 %
Risk-2 %
DWM-yield %
830
832
7.36
7.35
7.34
7.33
7.32
7.31
7.30
7.29
7.28
7.27
7.26
7.25
0.06
0.08
0.10
0.12
0.14
0.16
0.015
0.020
0.025
0.030
0.035
0.040
0.045
0.20
7.52
7.54
7.56
7.58
7.60
7.62

Fig. 17



812
840
814
840
B-yield %
7.345
7.340
7.335
7.330
7.325
7.320
7.315
7.310
7.305
7.300
7.295
0.09
0.10
0.11
0.12
0.13
0.14
Risk-1 %
0.020
0.025
0.030
0.035
0.040
Risk-2 %
Risk-2 %
0.038
0.036
0.034
0.032
0.030
0.028
0.026
0.024
0.022
0.020
840
7.54
7.55
7.56
7.57
7.58
7.59
7.60
7.61
DWM-yield %
840
816
818

Fig. 18

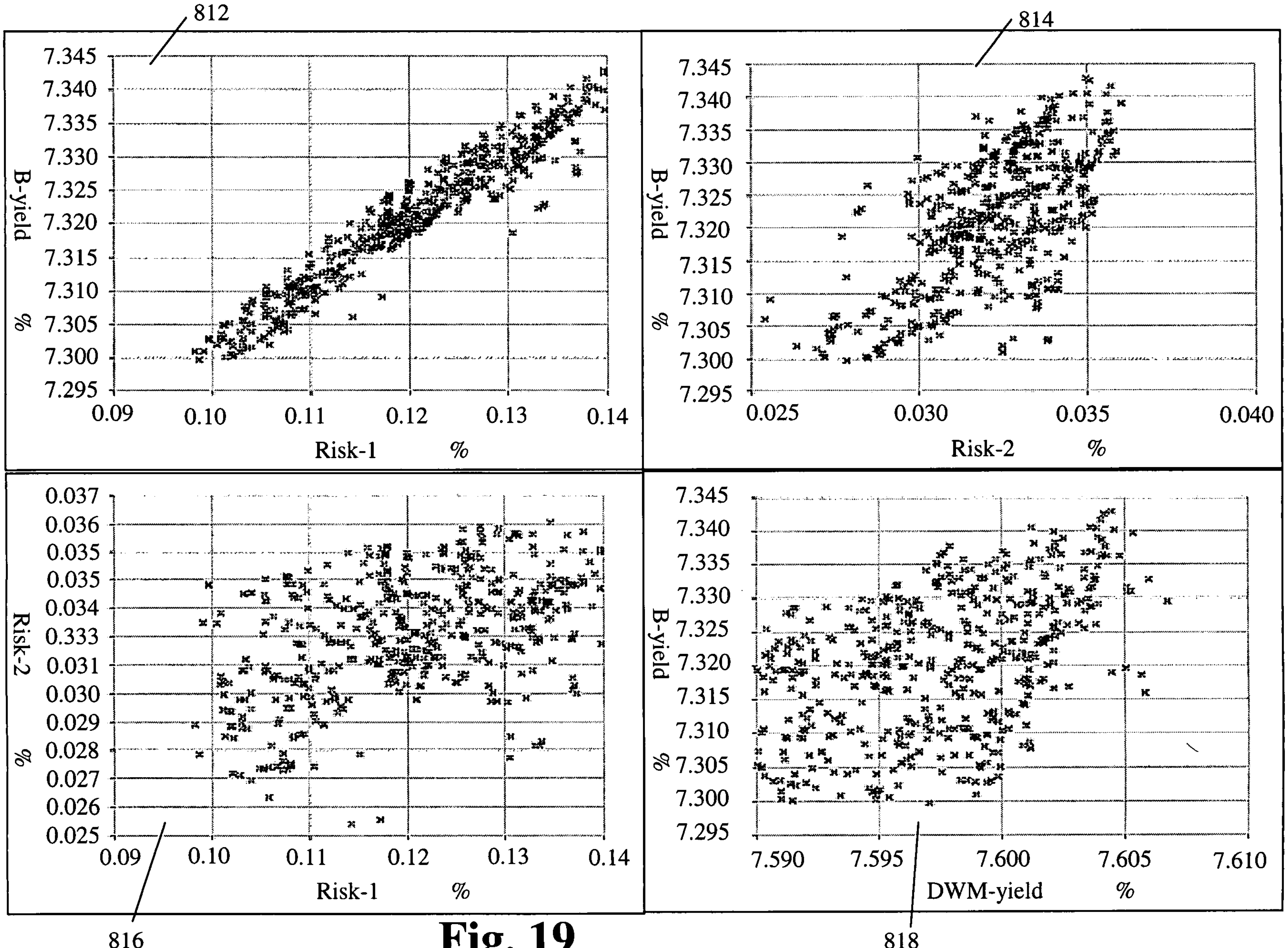
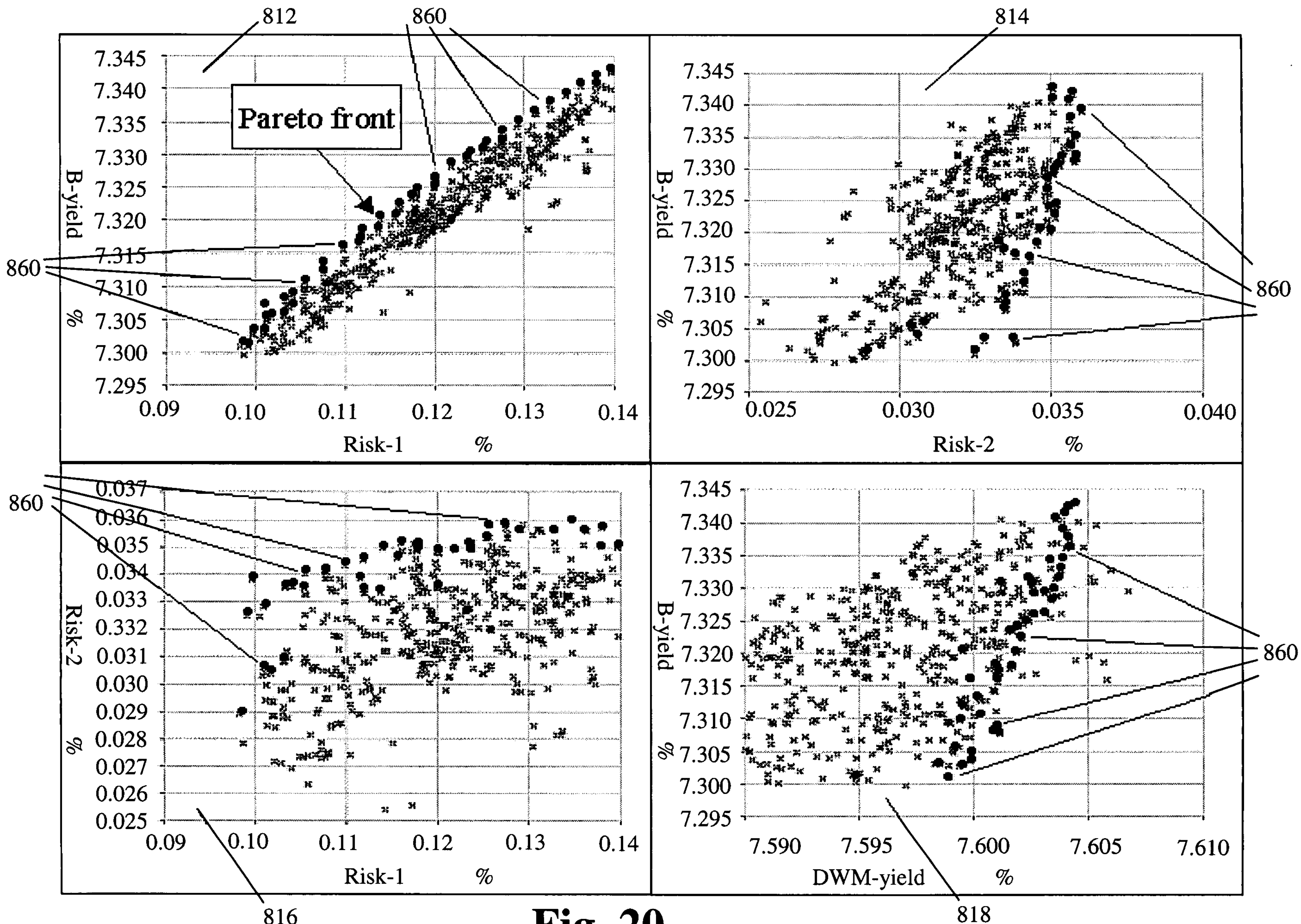

812
814
816
818
B-yield %
Risk-1 %
Risk-2 %
DWM-yield %
7.345
7.340
7.335
7.330
7.325
7.320
7.315
7.310
7.305
7.300
7.295
0.09
0.10
0.11
0.12
0.13
0.14
0.025
0.030
0.035
0.040
0.037
0.036
0.035
0.035
0.034
0.333
0.031
0.030
0.029
0.028
0.027
0.026
0.025
7.590
7.595
7.600
7.605
7.610

Fig. 19



812
860
Pareto front
7.345
7.340
7.335
7.330
7.325
7.320
7.315
7.310
7.305
7.300
7.295
B-yield %
0.09
0.10
0.11
0.12
0.13
0.14
Risk-1 %
814
B-yield %
0.025
0.030
0.035
0.040
Risk-2 %
816
0.037
0.036
0.035
0.034
0.033
0.332
0.031
0.030
0.029
0.028
0.027
0.026
0.025
Risk-2 %
818
7.590
7.595
7.600
7.605
7.610
DWM-yield %

Fig. 20

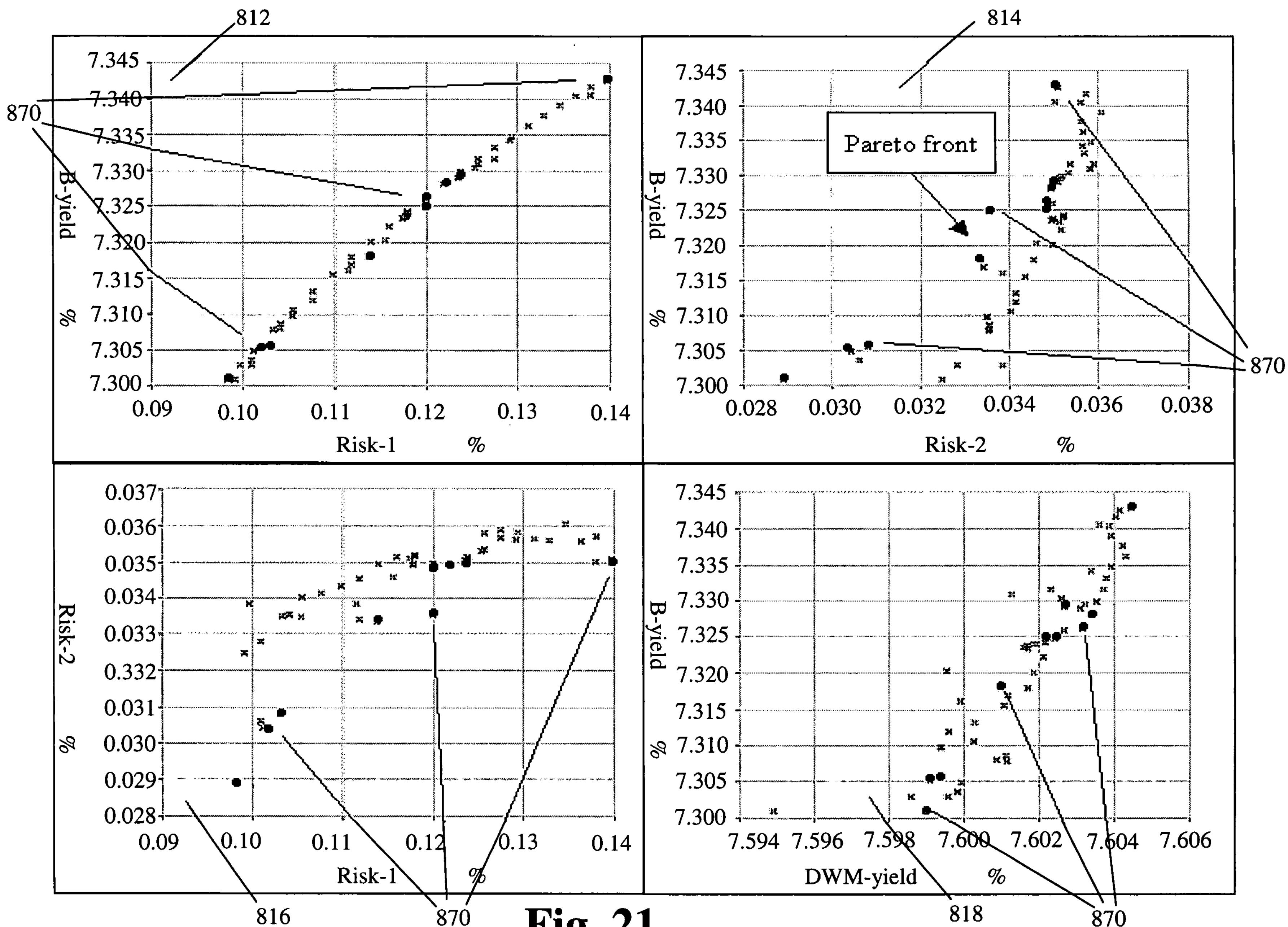

812
814
870
Pareto front
B-yield %
Risk-1 %
Risk-2 %
DWM-yield %
816
818
7.345
7.340
7.335
7.330
7.325
7.320
7.315
7.310
7.305
7.300
0.09
0.10
0.11
0.12
0.13
0.14
0.028
0.030
0.032
0.034
0.036
0.038
0.037
0.036
0.035
0.034
0.033
0.332
0.031
0.030
0.029
0.028
7.594
7.596
7.598
7.600
7.602
7.604
7.606

Fig. 21

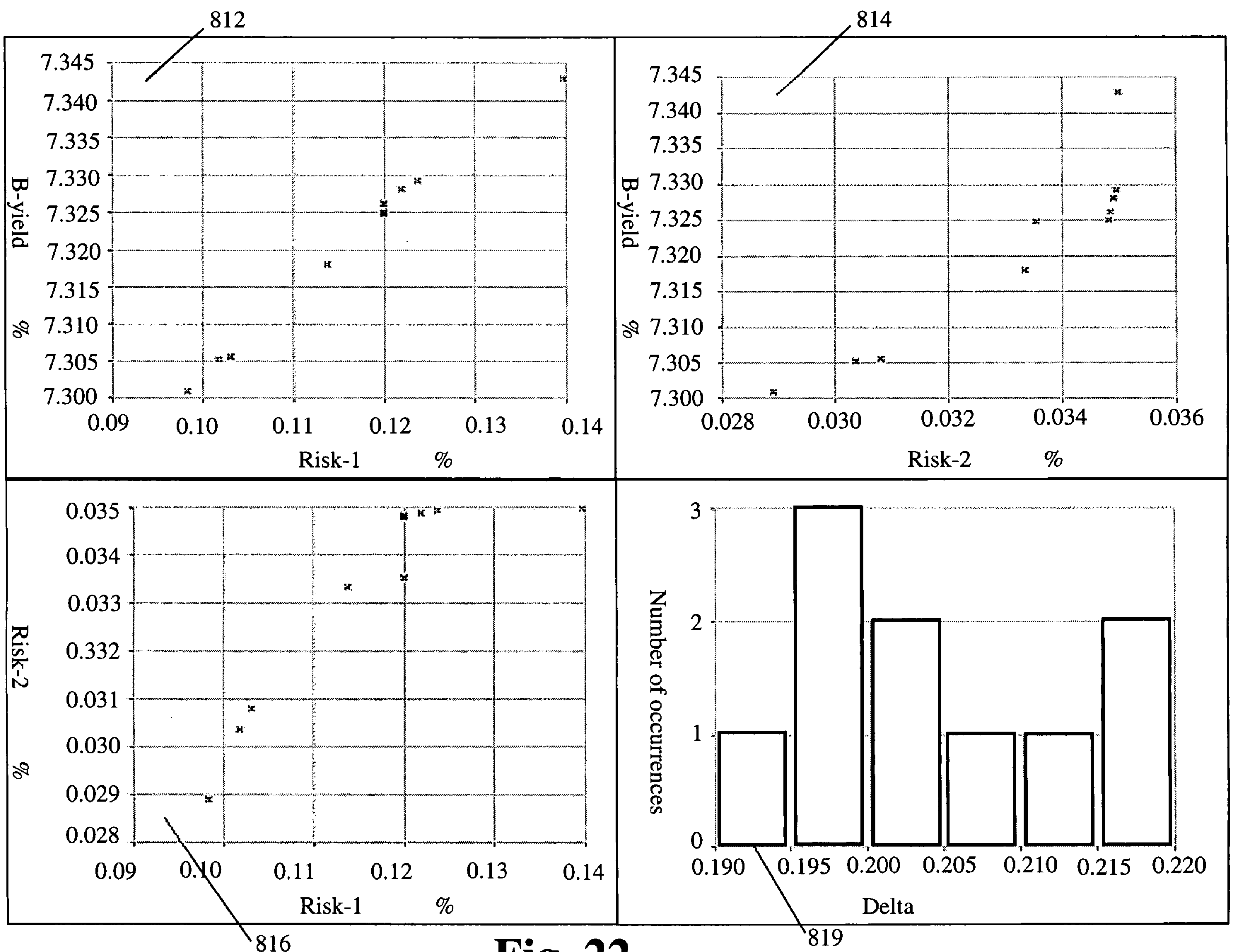
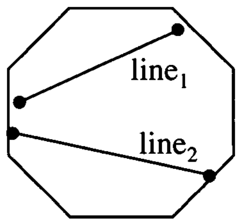
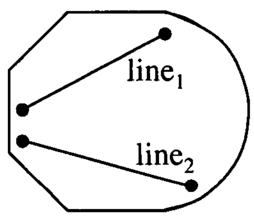
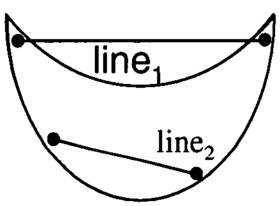

812
814
816
819
B-yield %
Risk-1 %
Risk-2 %
Number of occurrences
Delta
7.345
7.340
7.335
7.330
7.325
7.320
7.315
7.310
7.305
7.300
0.09
0.10
0.11
0.12
0.13
0.14
0.028
0.030
0.032
0.034
0.036
0.035
0.034
0.033
0.332
0.031
0.030
0.029
0.028
0.190
0.195
0.200
0.205
0.210
0.215
0.220

Fig. 22

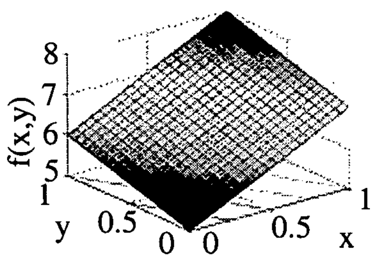
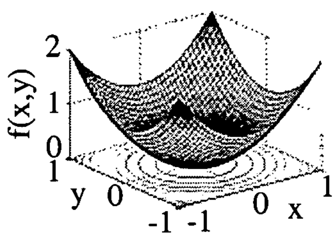
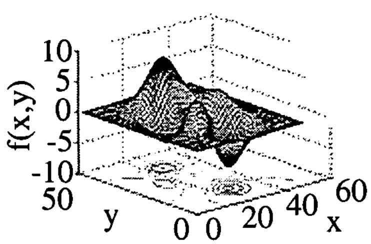
Feasible Regions for Optimization

Figure 33

Graphic Visual	Word Description	Example Equation	GEAM
Linear Convex Space 	• For any two points in the space, the line connecting the two points is **always contained** in the same space • Space is defined using **linear** equations	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{81} & a_{82} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_8 \end{bmatrix}$ **Set of linear equations**	• Market value weighted yield formulation • Duration weighted yield formulation
Nonlinear Convex Space 	• For any two points in the space, the line connecting the two points is **always contained** in the same space • Space is defined using some **nonlinear** equations	$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{51} & a_{52} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_5 \end{bmatrix}$ $x^2 + y^2 \leq \alpha$ **Nonlinear equation**	• Interest rate sigma formulation
Nonlinear Nonconvex Space 	• For any two points in the space, the line connecting the two points is <u>not</u> **always contained** in the same space • Space is defined using some **nonlinear** equations	$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x^2 \\ x \\ y \end{bmatrix} \leq \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ **Set of nonlinear equations**	• Interest rate sigma and VAR formulation • VAR is a nonlinear nonconvex constraint

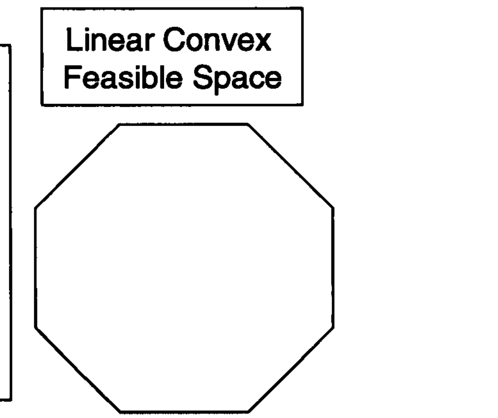
Objective Functions

Figure 34

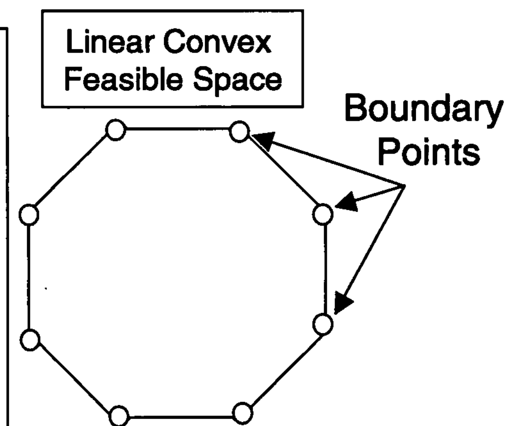
Graphic Visual	Word Description	Example Equation	GEAM
Linear Function 	• Function is defined using linear equations • Straightforward math relationship • **Easy to optimize**	$f(x, y) = 2x + y + 5$	• Market value weighted yield • Duration weighted yield
Nonlinear Convex Function 	• Function is defined using a **nonlinear** equation • Functional gradients lead to single optimum • **Harder to optimize**	$f(x, y) = x^2 + y^2$	• Interest rate sigma
Nonlinear Nonconvex Function 	• Function is defined using **complex nonlinear** equations • Multiple local optima • Functional gradients are inefficient • **Very hard to optimize**	$f(x,y) = g_1(x,y) + g_2(x,y) + g_3(x,y) + g_4(x,y)$	• Interest rate sigma and VAR

Evolutionary Search Augmented with Domain Knowledge

Multi-objective portfolio optimization problem is formulated as a problem with Multiple linear, nonlinear and nonlinear nonconvex objectives. However, the **domain knowledge** allows us to use strictly **linear and convex constraints.**



Knowledge about geometry of feasible space (i.e. convexity), allowed us develop a feasible space boundary sampling algorithm (**solutions archive generation**). By knowing the boundary of the search space, we can exploit that knowledge to design efficient interior sampling methods.



Convex crossover is a powerful interior sampling method, which is guaranteed to produce feasible offspring solutions. Given parents P_1, P_2, it creates offspring $O_1 = \lambda P_1 + (1-\lambda)P_2$, $O_2 = (1-\lambda)P_1 + \lambda P_2$. An offspring O_k and P_k can crossed over to produce more **diverse** offspring.

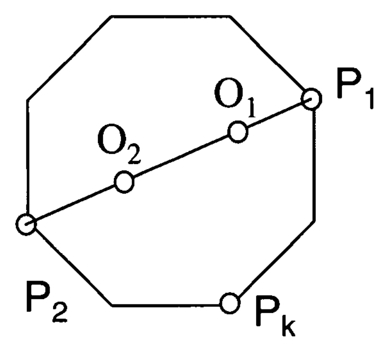


Figure 36